Many Body Localization

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Metal-insulator transition

in a

weakly interacting many-electron system

with localized single-particle states

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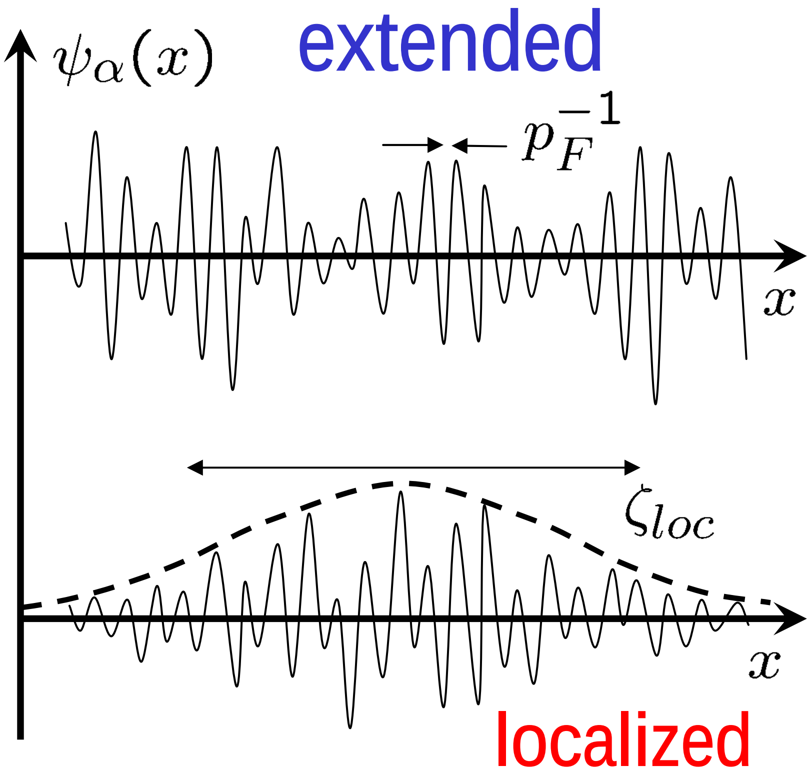
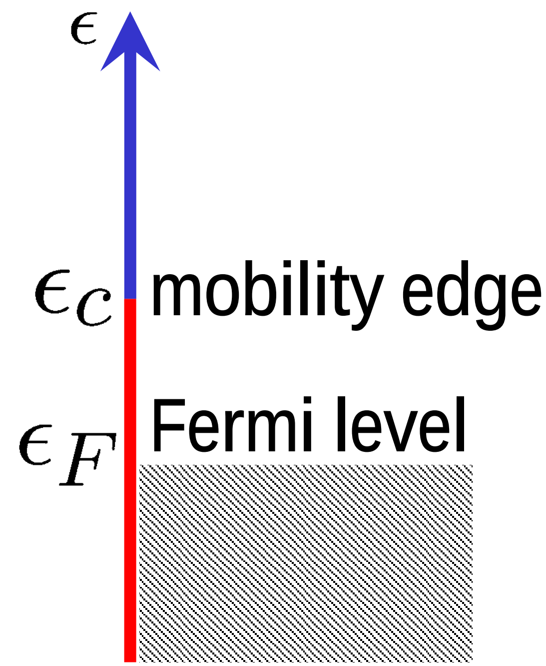
with localized single-particle states

Can hopping conductivity exist without phonons ?

Free electrons in a random potential





$$\epsilon_c < \infty \quad \Rightarrow \quad \sigma(T) \propto \exp\left(-\frac{\epsilon_c - \epsilon_F}{T}\right)$$

thermal population of extended eigenstates

$$\epsilon_c = \infty \quad \Rightarrow \quad \boxed{\sigma(T) = 0}$$

all eigenstates are localized

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

Absence of Diffusion in Certain Random Lattices

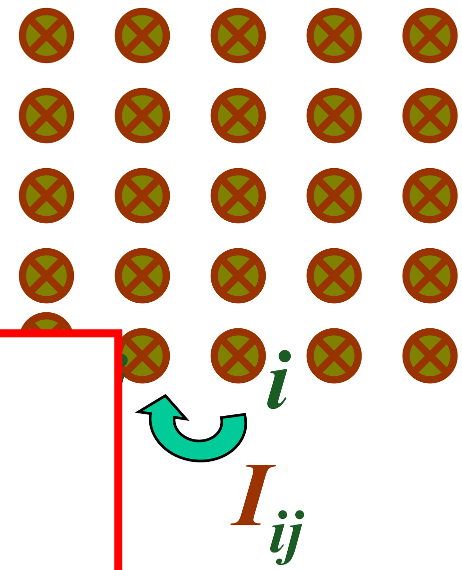
P. W. ANDERSON
Bell Telephone Laboratories, Murray Hill, New Jersey

(Received October 10, 1957)

This paper presents a simple model for such processes as spin diffusion or conduction in the "impurity band." These processes involve transport in a lattice which is in some sense random, and in them diffusion is expected to take place via quantum jumps between localized sites. In this simple model the essential randomness is introduced by requiring the energy to vary randomly from site to site. It is shown that at low enough densities no diffusion at all can take place, and the criteria for transport to occur are given.

Anderson Model

- *Lattice - tight binding model*
- *Onsite energies* ε_i *- random*
- *Hopping matrix elements* I_{ij}



In fact, i,j can be states
in <u>any space</u>
(not necessarily coordinate)

Anderson Transition:

$I < I_c$

Insulator

All eigenstates are localized

$I > I_c$

Metal

There appear states extended all over the system

At $I>I_c$ there will be always level mismatched from given by

$$|\varepsilon_i - \varepsilon_j| < I$$

and the resonance transport will occur

Free electrons + disorder:

$d=1$: all states are localized

$d=2$: the same

$d=3$: mobility edge

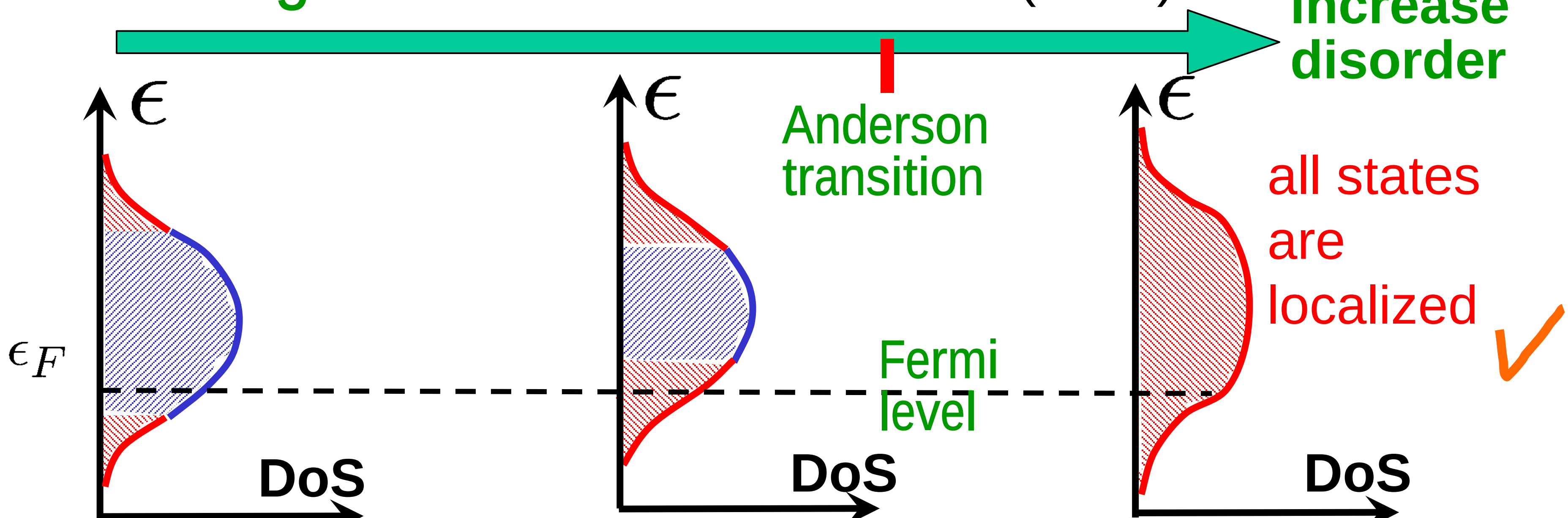
Free electrons + disorder:

d=1: all states are localized

d=2: the same

d=3: mobility edge

Single-band Anderson model ($d>2$)



$\sigma(T) = const$ $\quad$ $\sigma(T) \propto e^{-\frac{\epsilon_c - \epsilon_F}{T}}$ $\quad$ $\sigma(T) = 0$

Main assumption: all states are localized

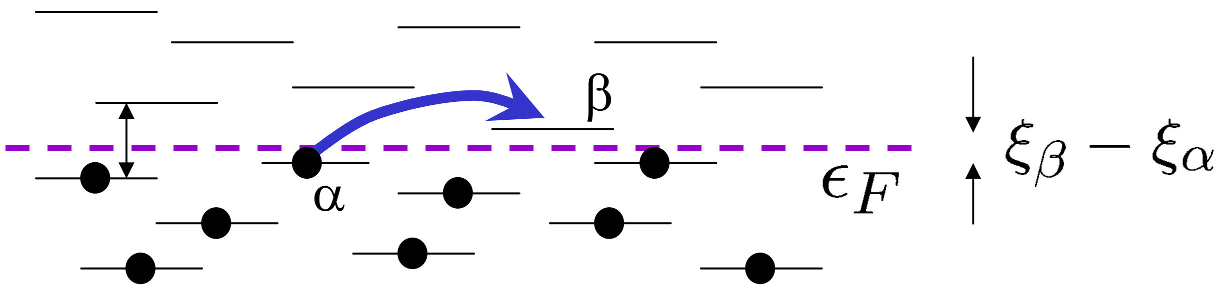
Beyond one-electron picture

Hopping Conductivity:

transitions between localized states due to inelastic processes

Beyond one-electron picture

Hopping Conductivity: transitions between localized states due to inelastic processes



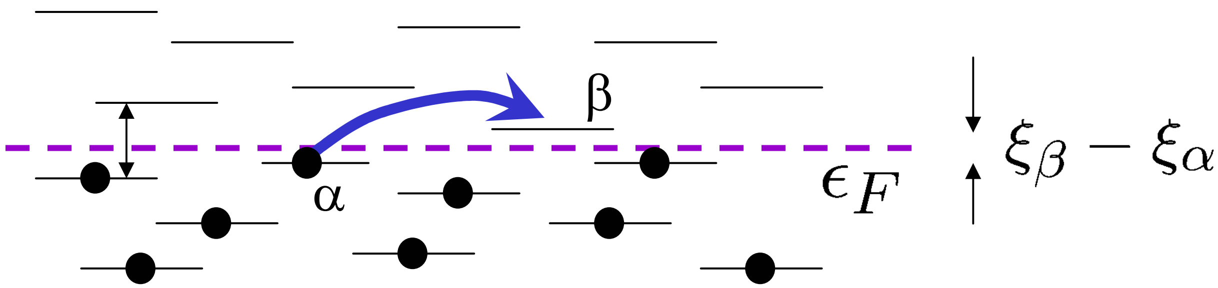
energy mismatch !

Need energy! $T = 0 \quad \Rightarrow \quad \sigma = 0$ (any bath!)

Bath? $T \to 0 \quad \Rightarrow \quad \sigma = ?$

Beyond one-electron picture

Hopping Conductivity: transitions between localized states due to inelastic processes



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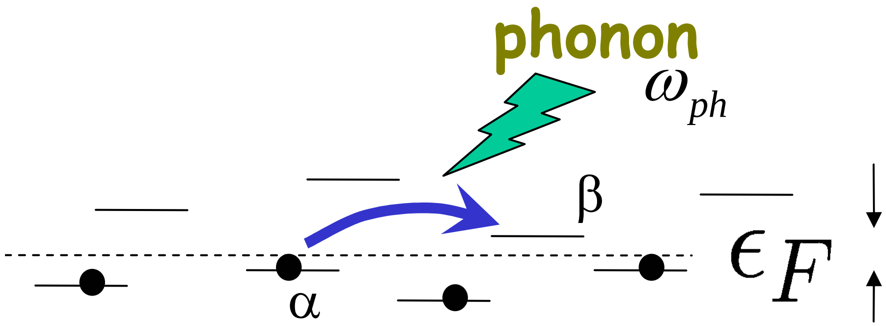
Bath? $T \to 0 \quad \Rightarrow \quad \sigma = ?$

$$\sigma(T) \propto \Gamma_\alpha$$ (inelastic lifetime)$^{-1}$

Phonon-induced hopping

Bath – Phonons

Continuous spectrum !



$$\xi_\beta - \xi_\alpha = \omega_{ph}$$

energy difference can always be matched by a phonon

Phonon-induced hopping

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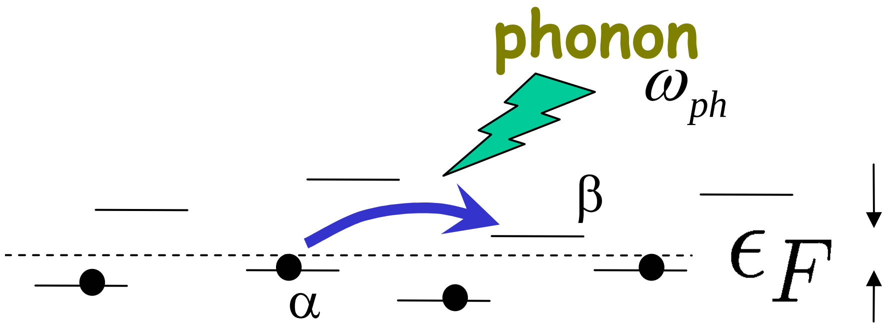
Continuous spectrum !



$$\xi_\beta - \xi_\alpha = \omega_{ph}$$

Mott's variable range hopping: $T = \delta_\zeta$

where: $\delta_\zeta \equiv \left(\nu \zeta_{loc}^d\right)$

ν electronic DoS:

ζ_{loc} localization length

d # of dimensions

$$\sigma(T) \propto T^\gamma \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{1/(d+1)}\right]$$

model-dependent prefactor

Universal for any bath, provided that it has a continuous spectrum of delocalized excitations down to zero energy and no Coulomb gap!

Can hopping conductivity exist **without phonons** ?

1. Temperature is finite
2. All one-particle states are localized
3. Electrons interact with each other
4. They are isolated from the outside world

Does DC conductivity vanish or it is finite. ?

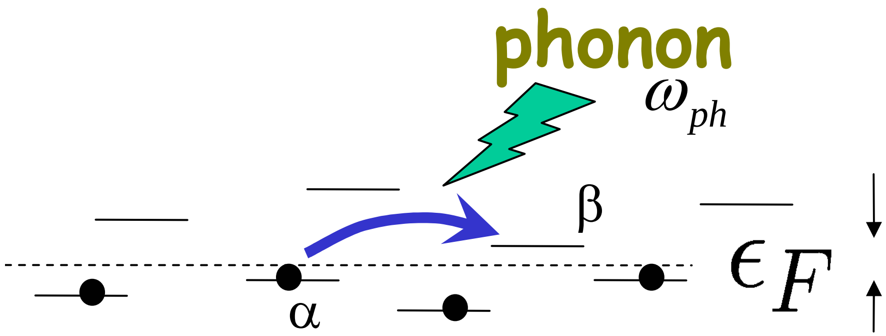
Q: Does AC conductivity vanish or it is finite ?

Q: Can plasmons (e-h excitations) play the role of phonons ?

Phonon-induced hopping

Bath – Phonons

Continuous spectrum !



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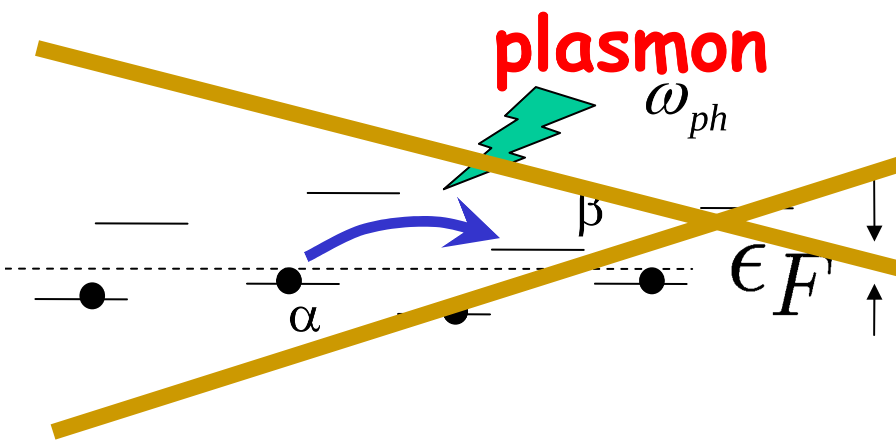


$$\xi_\beta - \xi_\alpha = \omega_{ph}$$

Plasmons are localized as well as electrons

Their spectrum is locally discrete

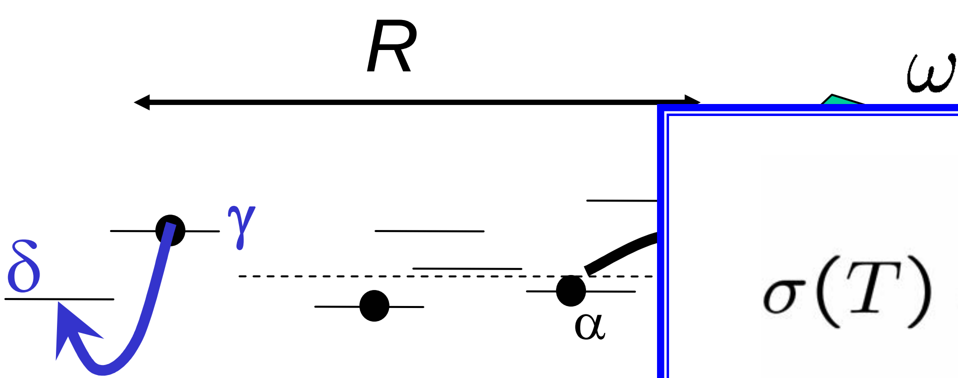
Q: Can we replace phonons with e-h pairs and obtain **phonon-less** *VRH*?

A#1: Sure [a person from the street (2005)]:

A#2: No way [L. Fleishman. P.W. Anderson (1980)]

(for Coulomb interaction in 3D — may be)

$$R \to \infty$$ ***Thus, the matrix element vanishes !!!***



$$\sigma(T) \propto \boxed{0\ *} \exp\left[-\left(\frac{\delta_\zeta}{T}\right)^{\frac{1}{d+1}}\right]$$

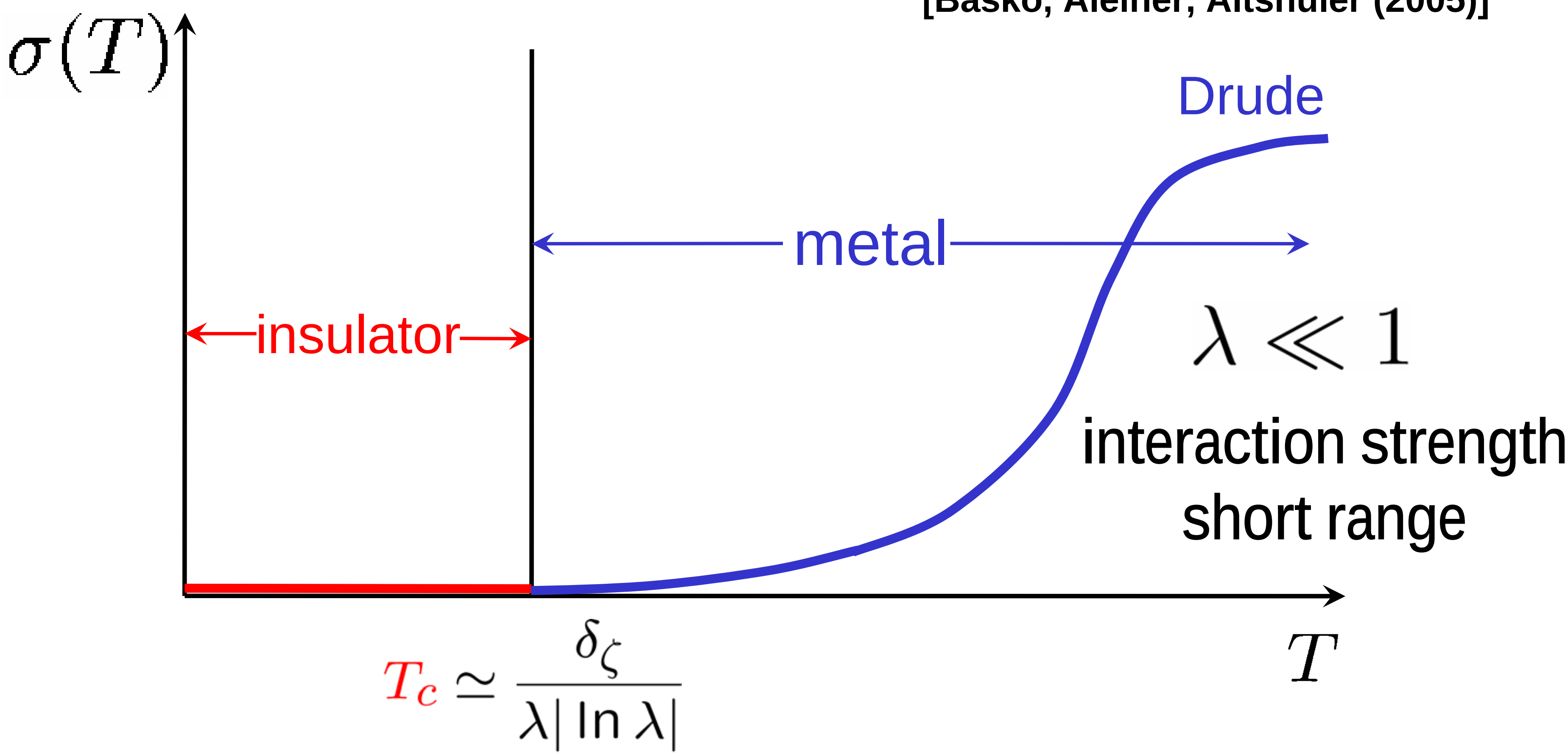
Q: Can we replace phonons with e-h pairs and obtain **phonon-less** *VRH?*

A#1: Sure [a person from the street (2005)]:

A#2: No way [L. Fleishman. P.W. Anderson (1980)]

A#3: Finite T ***Metal-Insulator Transition***

[Basko, Aleiner, Altshuler (2005)]



Finite temperature metal-insulator transition without changing any spatial symmetry

!

$\sigma = 0$ Physics: Many-excitations excitations turn out to be localized in the Fock space

VOLUME 78, NUMBER 14 PHYSICAL REVIEW LETTERS 7 APRIL 1997

Quasiparticle Lifetime in a Finite System: A Nonperturbative Approach

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[2]*Department of Condensed Matter Physics, The Weizmann Institute of Science, Rehovot, 76100, Israel*
[3]*Massachusetts Institute of Technology, 12-112, Cambridge, Massachusetts 02139*
(Received 30 August 1996)

The problem of electron-electron lifetime in a quantum dot is studied beyond perturbation theory by mapping onto the problem of localization in the Fock space. Localized and delocalized regimes are identified, corresponding to quasiparticle spectral peaks of zero and finite width, respectively. In the localized regime, quasiparticle states are single-particle-like. In the delocalized regime, each eigenstate is a superposition of states with very different quasiparticle content. The transition energy is $\epsilon_c \simeq \Delta(g/\ln g)^{1/2}$, where Δ is mean level spacing, and g is the dimensionless conductance. Near ϵ_c there is a broad critical region not described by the golden rule. [S0031-9007(97)02895-0]

Anderson localization in the many-body Fock space

$$\xi_\alpha \Rightarrow \xi_\alpha + \xi_\beta - \xi_\gamma \Rightarrow \xi_\alpha + \xi_\beta - \xi_\gamma + \xi_\delta - \xi_\varphi \Rightarrow \ldots.$$

1 3 5 ….

many-body Fock states → sites with random energies

e-e interaction → coupling between sites

metal-insulator transition → Anderson transition

$$\sigma(T) \propto \Gamma_\alpha$$ (inelastic lifetime)$^{-1}$

Starting Point: Disorder + Interaction

$$\hat{H} = \hat{H}_0 + \hat{V}_{int};$$

$$\hat{H}_0 = \sum_\alpha \xi_\alpha \hat{c}^\dagger_\alpha \hat{c}_\alpha;$$

$$\hat{V}_{int} = \frac{1}{2}\sum_{\alpha\beta\gamma\delta} V_{\alpha\beta\gamma\delta}\hat{c}^\dagger_\alpha \hat{c}^\dagger_\beta \hat{c}_\gamma \hat{c}_\delta;$$

$$V_{\alpha\beta\gamma\delta} = \frac{1}{2}\int V(\boldsymbol{r},\boldsymbol{r}')\varrho_{\alpha\delta}(\boldsymbol{r})\varrho_{\beta\gamma}(\boldsymbol{r}')d\boldsymbol{r}d\boldsymbol{r}';$$

$$\varrho_{\alpha\beta}(\boldsymbol{r}) = \phi^*_\alpha(\boldsymbol{r})\phi_\delta(\boldsymbol{r}),$$

$$\left[-\frac{\boldsymbol{\nabla}^2}{2m} + U(\boldsymbol{r}) - \epsilon_F\right]\phi_\alpha(\boldsymbol{r}) = \xi_\alpha\phi_\alpha(\boldsymbol{r})$$

$\phi_\alpha(\vec{r})$ **localized single-particle eigenfunctions**

ζ **localization length**

Main energy scale

$$\delta_\zeta \equiv \frac{1}{\nu\zeta^d}$$

Energy spacing between the states localized nearby

ν **one-electron density of states**

We have to take into account that

1. A one-electron wave function decays exponentially as a function of the distance from its center.
2. There is level repulsion for the states localized nearby
3. Matrix elements of the interaction decay (probably as a power law) when differences between the energies of involved quasiparticles is increased.
4. These matrix elements have random sign.

Main energy scale

$$\delta_\zeta \equiv \frac{1}{\nu \zeta^d}$$

Energy spacing between the states localized nearby

ζ **localization length**

ν **one-electron density of states**

Need a model with small parameters

Main energy scale

$$\delta_\zeta \equiv \frac{1}{\nu\zeta^d}$$

Energy spacing between the states localized nearby

ζ **localization length**

ν **one-electron density of states**

Model with small parameters

Interesting physics at $T ? \ \delta_\zeta$

Weak short-range *e-e* interaction

$$V(\vec{r}-\vec{r}\,') = \frac{\lambda}{\nu}\delta(\vec{r}-\vec{r}\,')$$

DoS
per unit volume

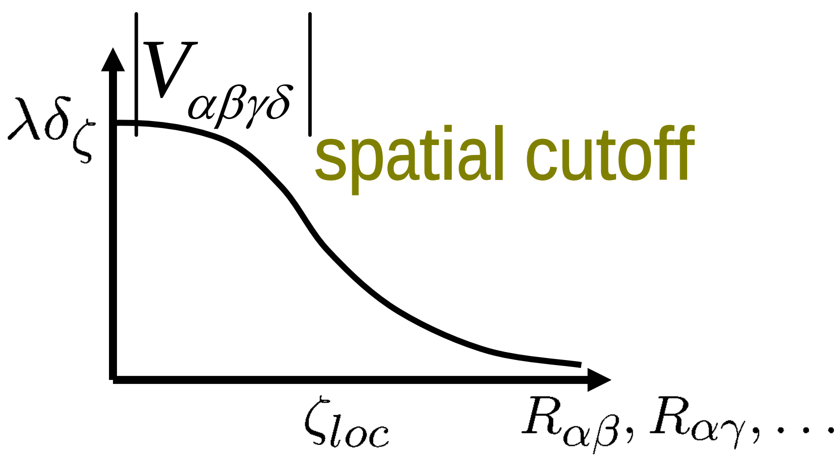
dimensionless
interaction
strength (<<1)

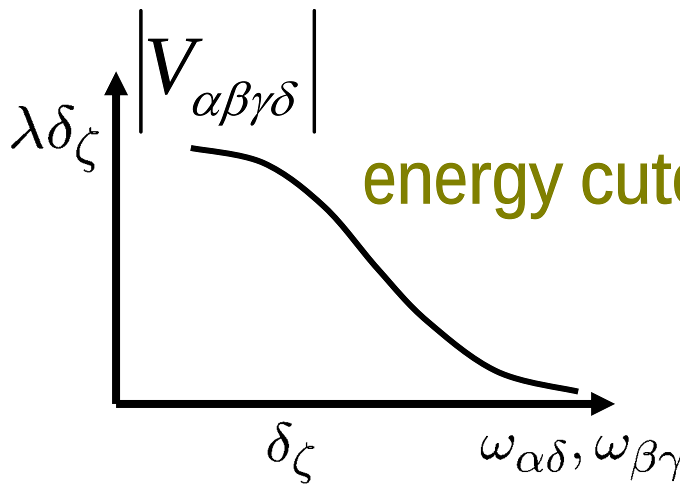
$$\frac{1}{\nu} = \delta_\zeta \zeta_{loc}^d$$

level
spacing

localization
volume

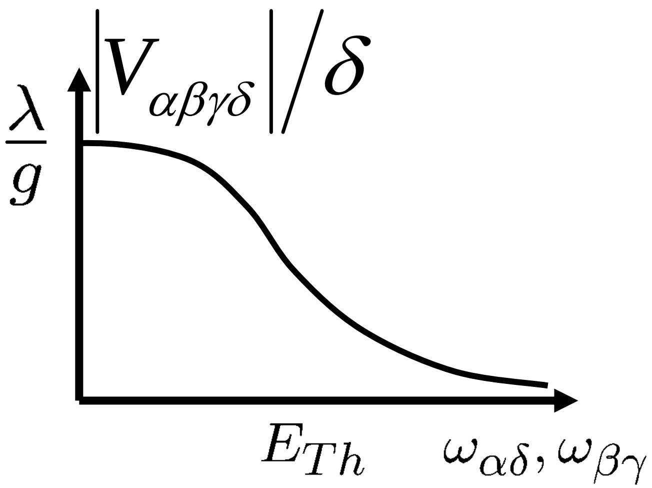
Matrix elements between localized wave functions:

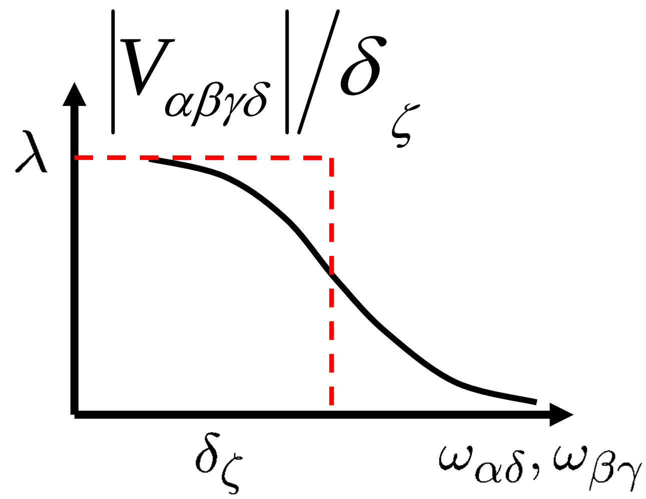




Energy cutoff

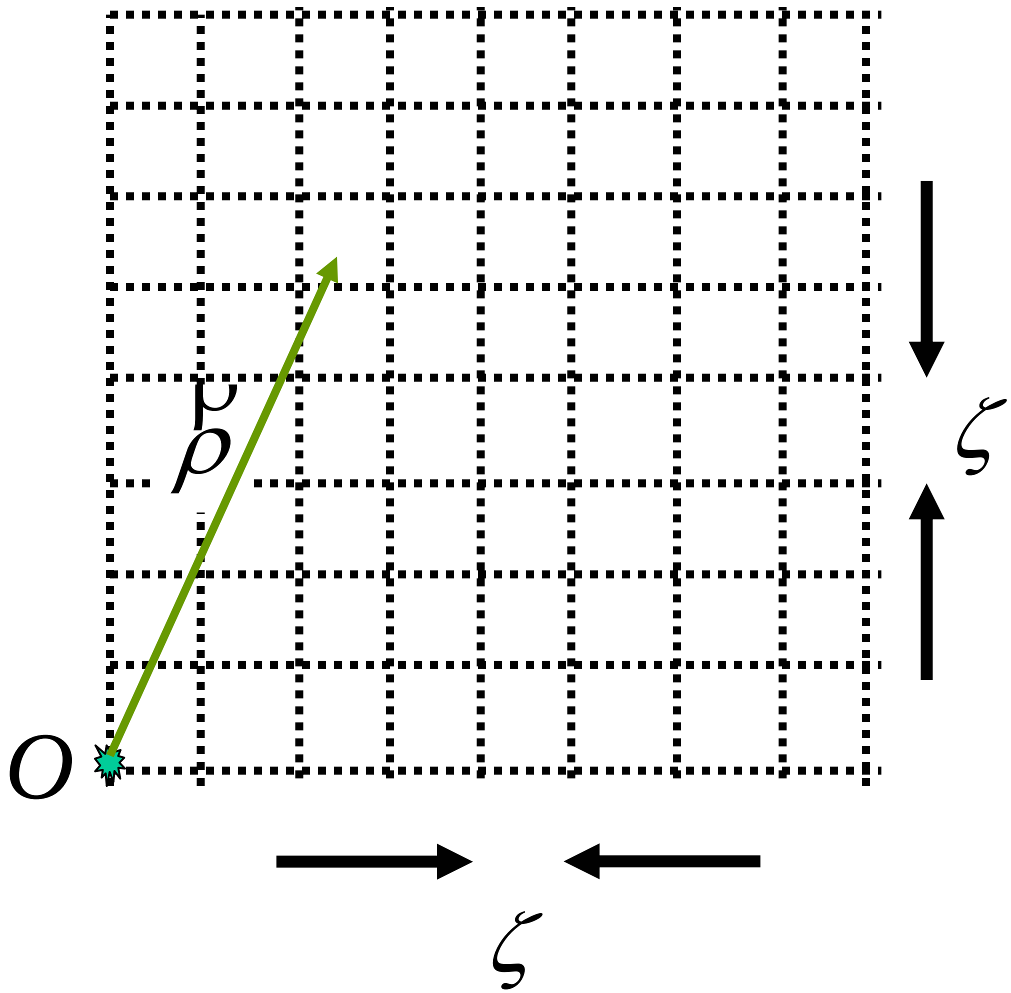
compare



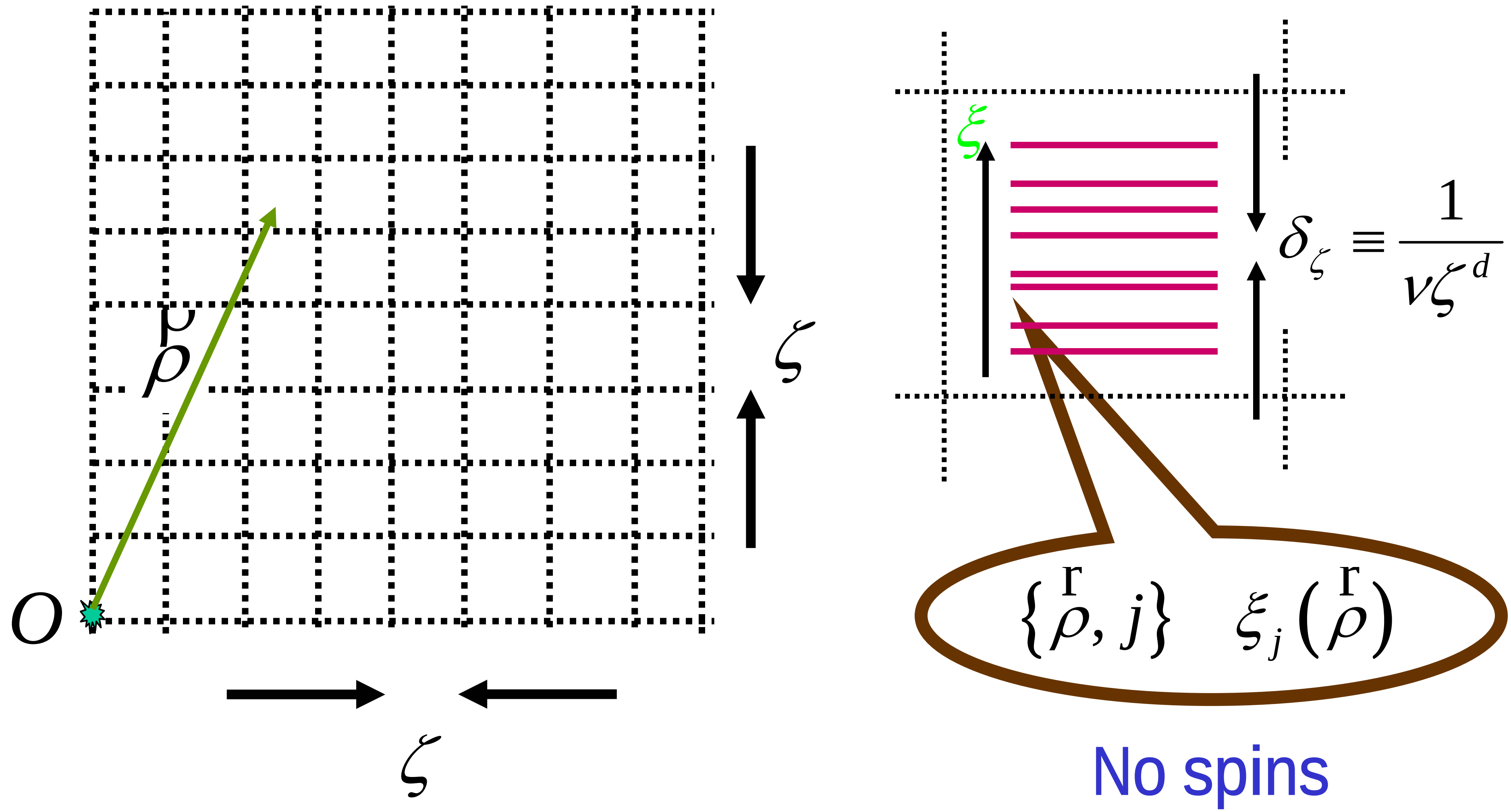


Quantum dot $g \equiv \frac{E_T}{\delta} ? \ 1$

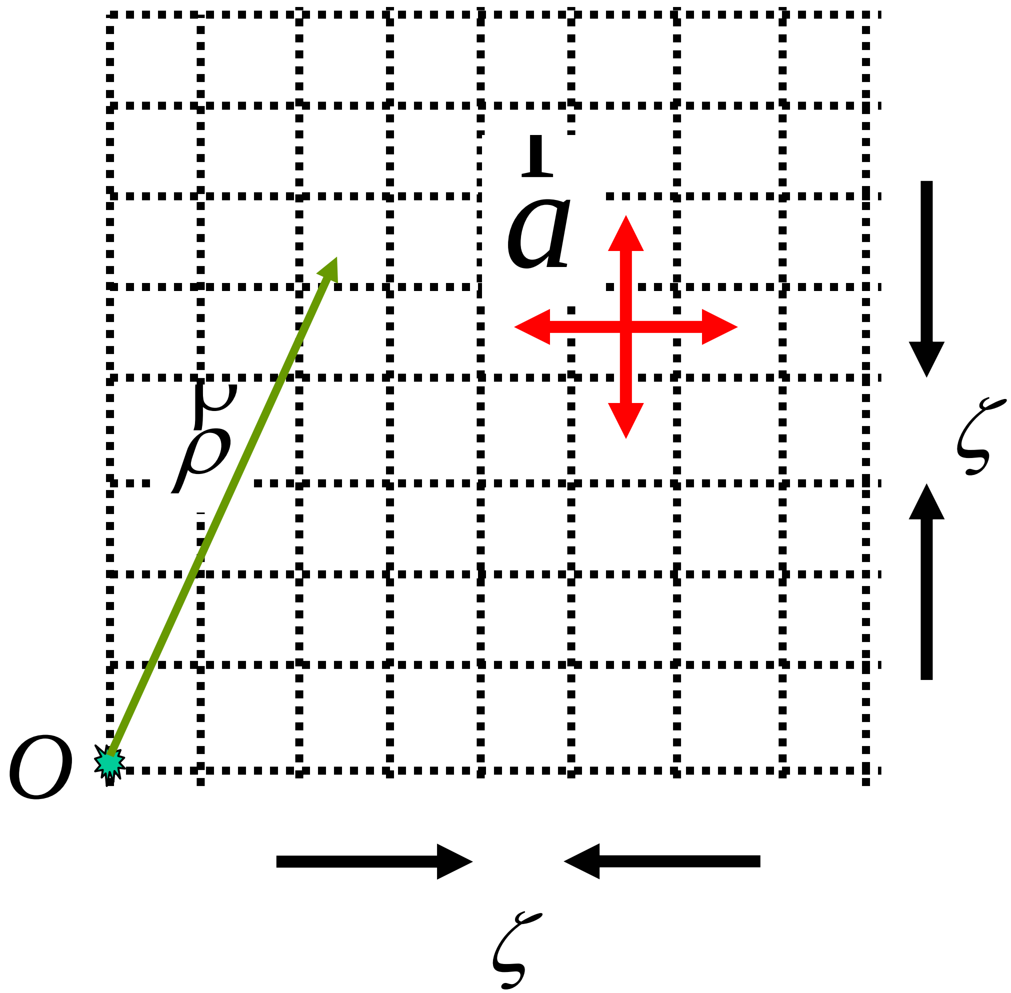
Localization volume $g : 1$

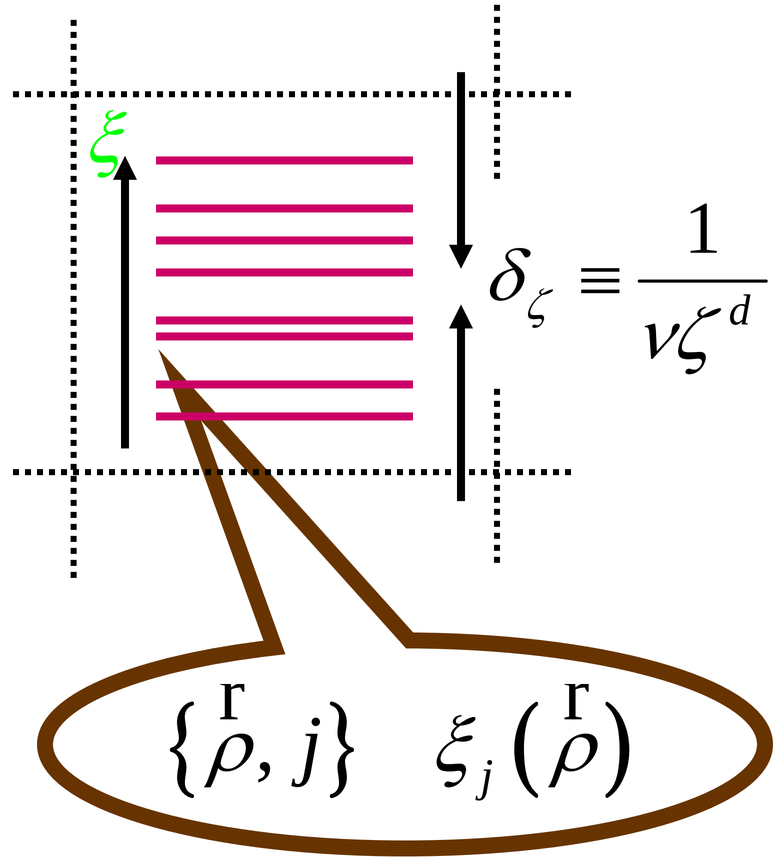


ρ
O
ζ
ζ


O
$\vec{\rho}$
ζ
ζ
ξ
$\delta_\zeta \equiv \dfrac{1}{\nu \zeta^d}$
$\{\vec{\rho}, j\} \quad \xi_j(\vec{\rho})$

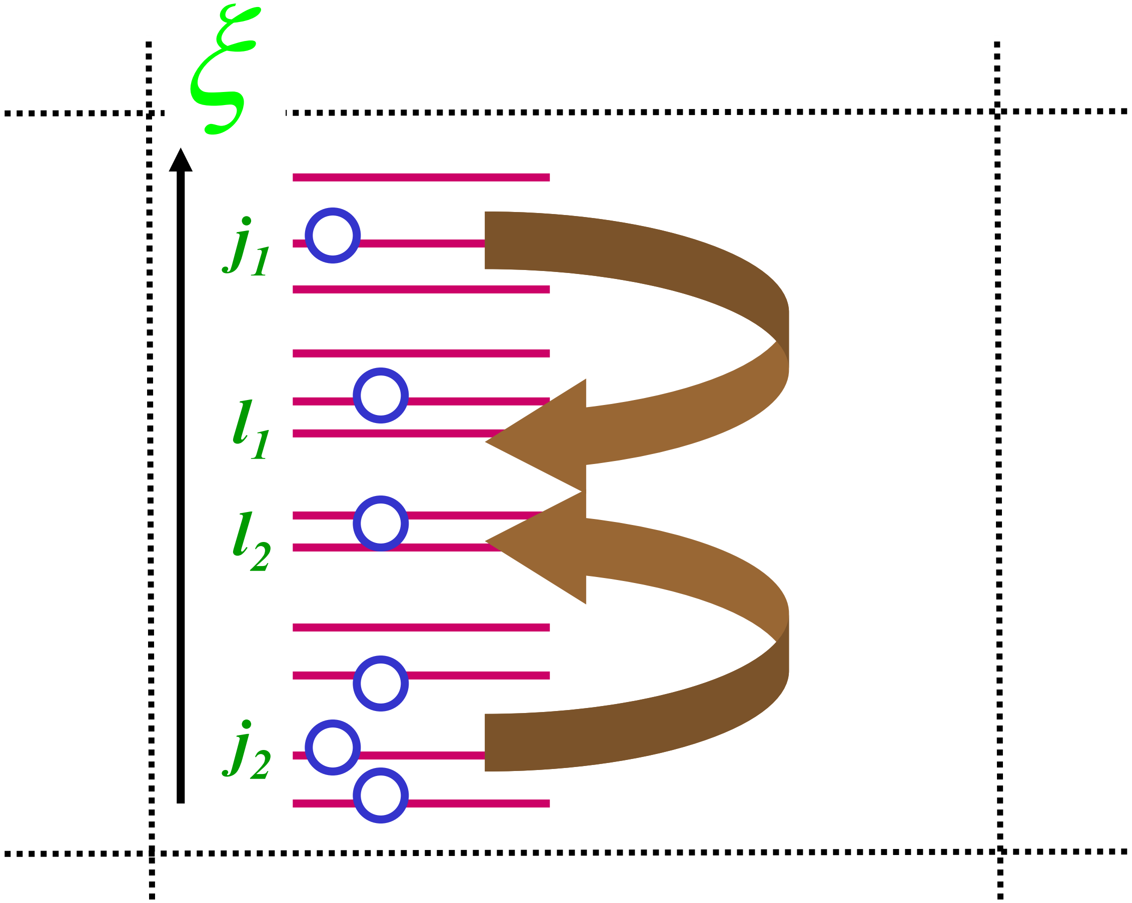
No spins



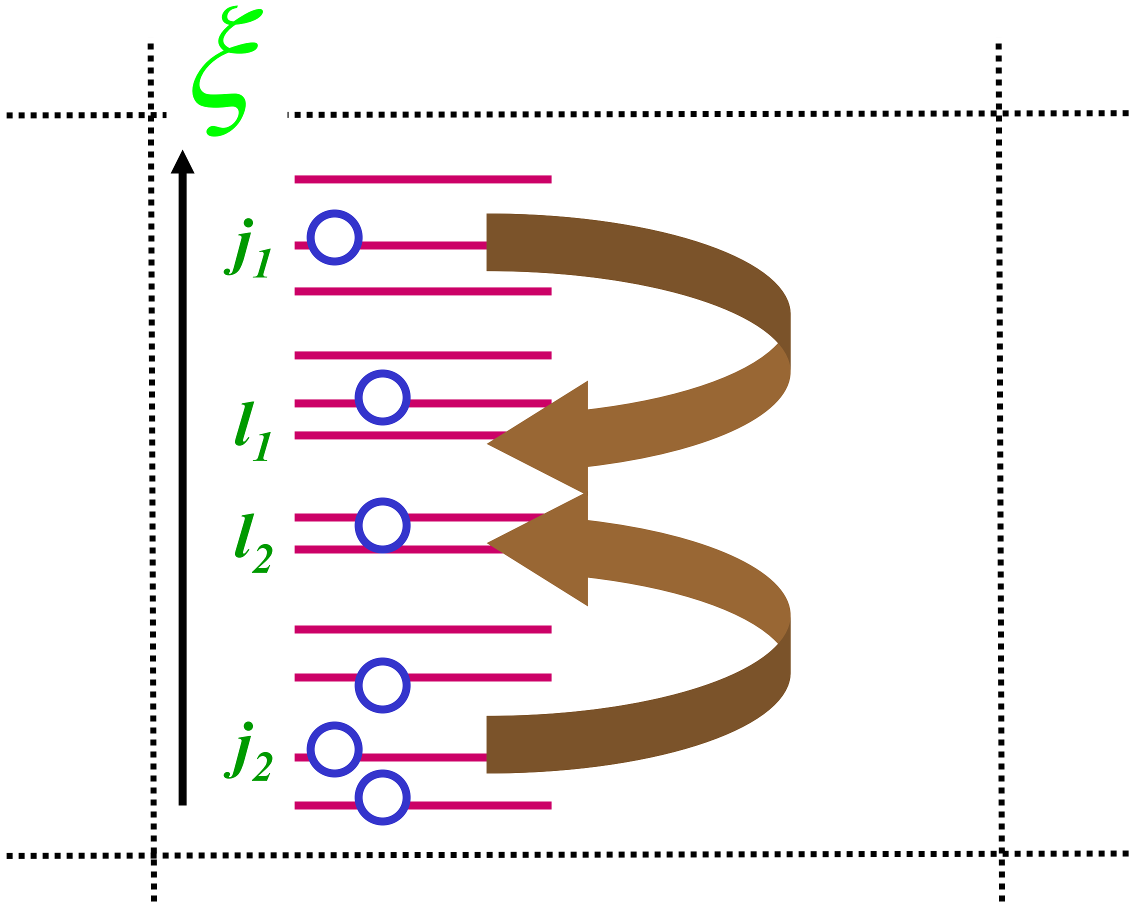


No spins

$$\boxed{\hat{H}_0 = \sum_{\vec{\rho},l} \hat{c}_l^\dagger(\vec{\rho}) \left[\xi_l(\vec{\rho}) \hat{c}_l(\vec{\rho}) + I\delta_\zeta \sum_{\vec{a},m} \hat{c}_m(\vec{\rho}+\vec{a}) \right]}$$



ξ
j_1
l_1
l_2
j_2



$$\boxed{\hat{V}_{\text{int}} = \frac{1}{2} \sum_{\vec{\rho}; l_1, l_2; j_1, j_2} V_{l_1, l_2}^{j_1, j_2}(\vec{\rho})\, \hat{c}_{l_1}^{\dagger}(\vec{\rho})\, \hat{c}_{l_2}^{\dagger}(\vec{\rho})\, \hat{c}_{j_1}(\vec{\rho})\, \hat{c}_{j_2}(\vec{\rho})}$$

Interaction only within the same cell;
no diagonal matrix elements

$$\hat{H}_0 = \sum_{\vec{\rho},l} \hat{c}_l^{\dagger}(\vec{\rho}) \left[\xi_l(\vec{\rho})\hat{c}_l(\vec{\rho}) + I\delta_{\zeta} \sum_{\vec{a},m} \hat{c}_m(\vec{\rho}+\vec{a}) \right]$$

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Effective Anderson Model?

Not yet :

What do we know about matrix elements?

$$\hat{H}_0 = \sum_{\vec{\rho},l} \hat{c}_l^\dagger(\vec{\rho}) \left[\xi_l(\vec{\rho})\hat{c}_l(\vec{\rho}) + I\delta_\zeta \sum_{\vec{a},m} \hat{c}_m(\vec{\rho}+\vec{a}) \right]$$

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Effective Anderson Model?

Not yet :

What do we know about matrix elements?

$$V_{l_1,l_2}^{j_1,j_2} = \lambda\delta_\zeta \frac{\sigma_{l_1}^{j_1}\sigma_{l_2}^{j_2}}{2} \Upsilon\left(\frac{\xi_{j_1}-\xi_{l_1}}{\delta_\zeta}\right)\Upsilon\left(\frac{\xi_{j_2}-\xi_{l_2}}{\delta_\zeta}\right) - (l_1 \leftrightarrow l_2)$$

$$\Upsilon(x) = \theta\left(\frac{M}{2} - |x|\right); \quad 1 = M < \frac{1}{\sqrt{\lambda}}$$

σ_l^j Random signs

$$\hat{H}_0 = \sum_{\vec{\rho},l} \hat{c}_l^{\dagger}(\vec{\rho}) \left[\xi_l(\vec{\rho})\, \hat{c}_l(\vec{\rho}) + I \delta_{\zeta} \sum_{\vec{a},m} \hat{c}_m(\vec{\rho}+\vec{a}) \right]$$

$$\hat{V}_{\mathrm{int}} = \frac{1}{2} \sum_{\vec{\rho};l_1,l_2;j_1,j_2} V_{l_1,l_2}^{j_1,j_2}(\vec{\rho})\, \hat{c}_{l_1}^{\dagger}(\vec{\rho})\, \hat{c}_{l_2}^{\dagger}(\vec{\rho})\, \hat{c}_{j_1}(\vec{\rho})\, \hat{c}_{j_2}(\vec{\rho})$$

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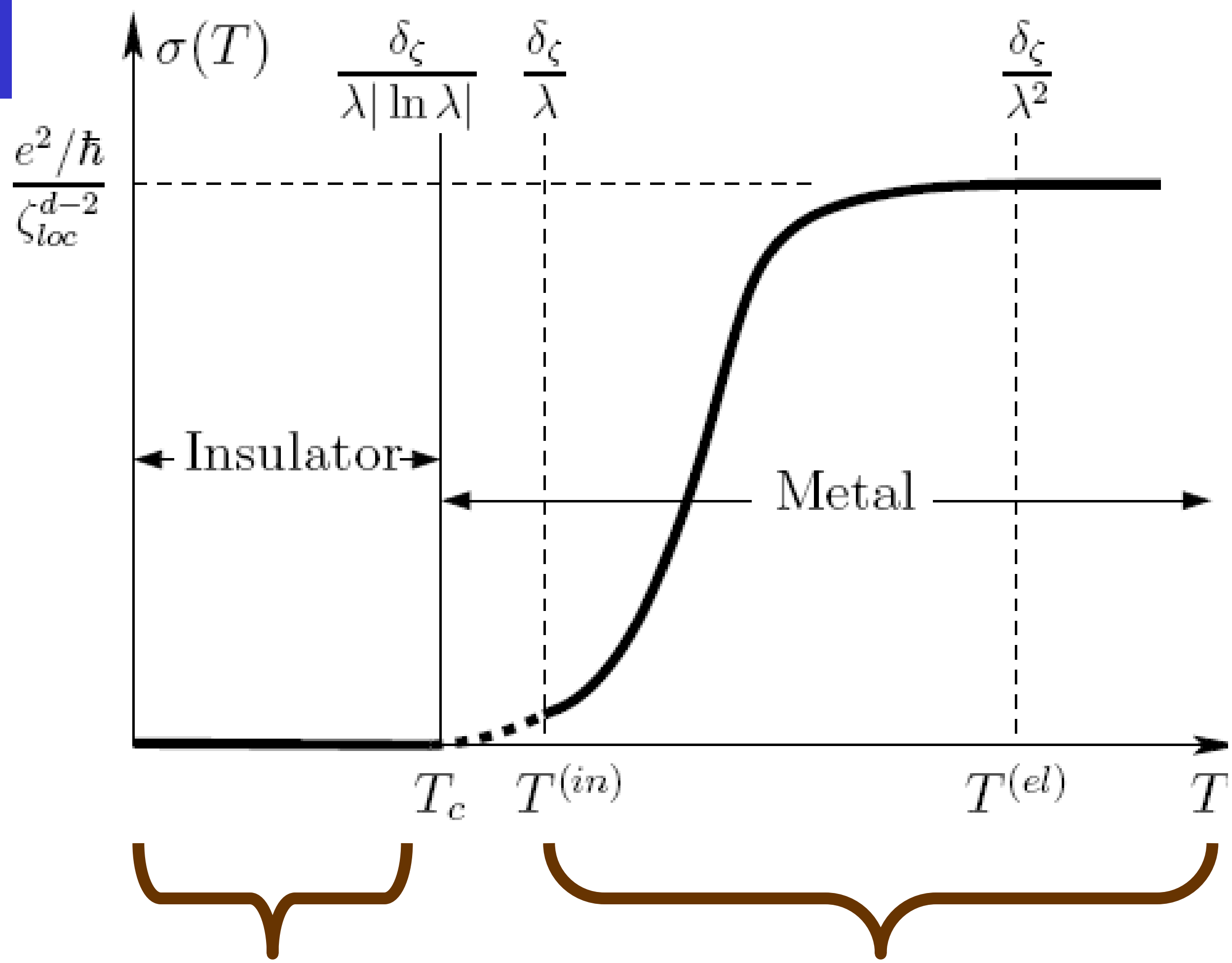
$$\Upsilon(x) = \theta\left(\frac{M}{2} - |x| \right); \quad 1 = M < \frac{1}{\sqrt{\lambda}}$$

Parameters: $\lambda, I, M^{-1} = 1$

Weakly connected grains?
Different problem?

correct behavior of the <u>tails</u>
of one particle wavefunctions

Technique



Self-Consistent Born Approximation

Boltzmann Equation

PHYSICAL REVIEW VOLUME 109, NUMBER 5 MARCH 1, 1958

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J. Phys. C: Solid State Phys., Vol. 6, 1973. Printed in Great Britain. © 1973

A selfconsistent theory of localization

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‡ Cavendish Laboratory, Cambridge, England and Bell Laboratories, Murray Hill, New Jersey, 07974, USA

Received 12 January 1973

Abstract. A new basis has been found for the theory of localization of electrons in disordered systems. The method is based on a selfconsistent solution of the equation for the self energy in second order perturbation theory, whose solution may be purely real almost everywhere (localized states) or complex everywhere (nonlocalized states). The equations used are exact for a Bethe lattice. The selfconsistency condition gives a nonlinear integral equation in two variables for the probability distribution of the real and imaginary parts of the self energy. A simple approximation for the stability limit of localized states gives Anderson's 'upper limit approximation'. Exact solution of the stability problem in a special case gives results very close to Anderson's best estimate. A general and simple formula for the stability limit is derived; this formula should be valid for smooth distribution of site energies away from the band edge. Results of Monte Carlo calculations of the selfconsistency problem are described which confirm and go beyond the analytical results. The relation of this theory to the old Anderson theory is examined, and it is concluded that the present theory is similar but better.

Idea of the calculation:

1. Start with some infinitesimal width Γ_b (***Im*** part of the self-energy due to a bath) of each one-electron eigenstate
2. Consider ***Im*** part of the self-energy Γ in the presence of tunneling and ***e-e*** interaction.
3. Calculate the probability distribution function $P(\Gamma)$
4. Consider the limit: $\lim_{\Gamma_b \to 0; \Omega \to \infty} P(\Gamma) \equiv P_0(\Gamma)$

Ω is the volume of the system

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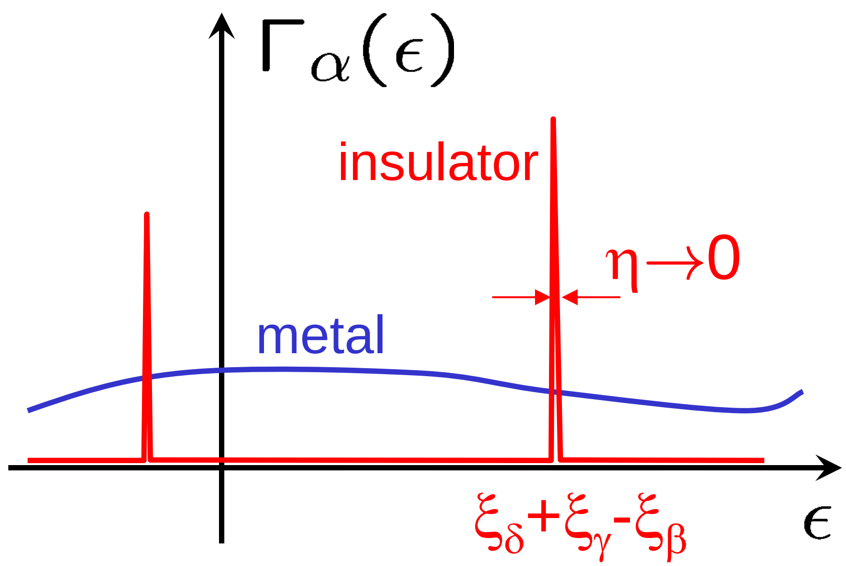
Ω is the volume of the system

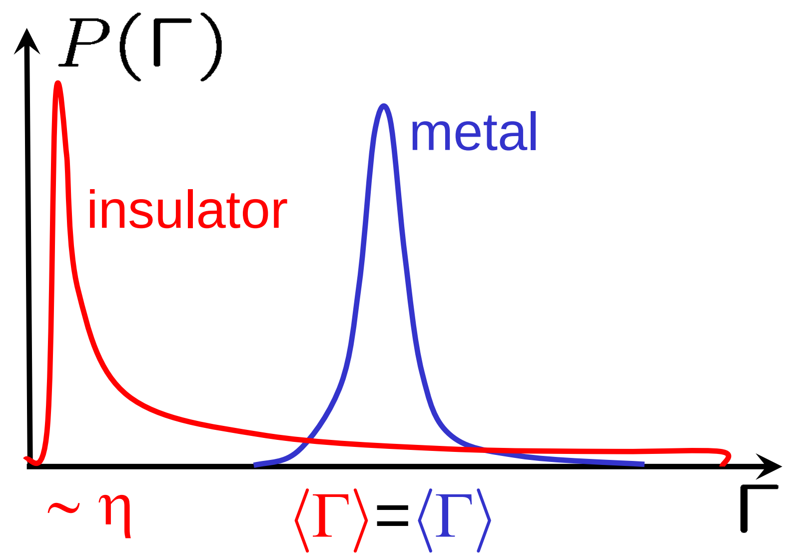
$$P_0(\Gamma) \begin{cases} = \delta(\Gamma) & \text{- insulator} \\ \neq 0 \ for\, \Gamma \neq 0 & \text{- metal} \end{cases}$$

What to calculate? **(Anderson, 1958)**

$$\Gamma_\alpha(\epsilon) = \mathrm{Im}\,\Sigma_\alpha(\epsilon + i\eta)$$ – random quantity



behavior for a
given realization



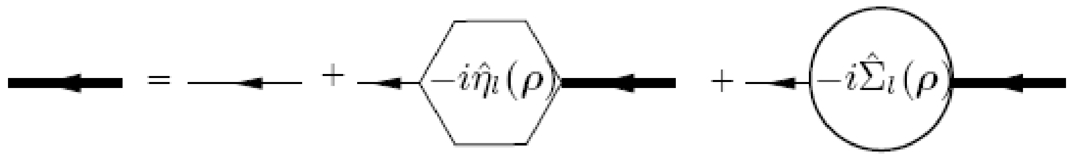
probability distribution
for a fixed energy

working criterion:

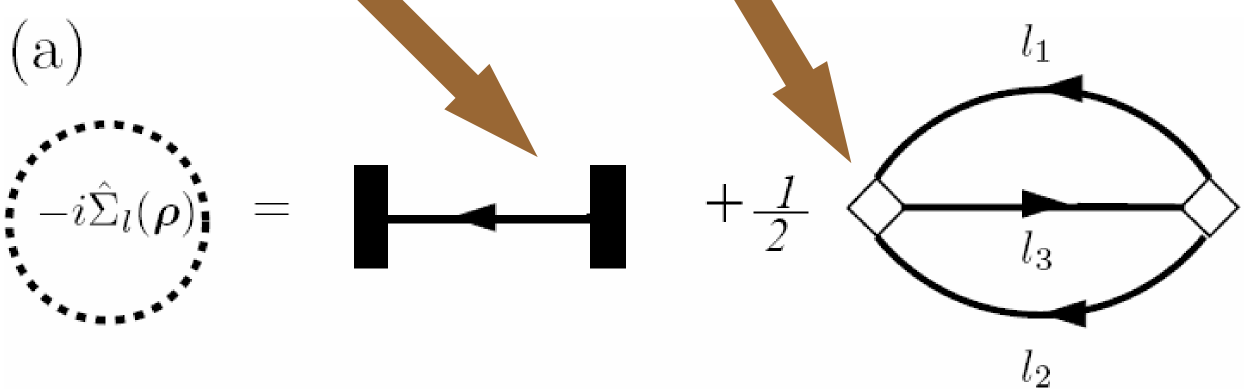
$$\lim_{\eta\to 0}\lim_{\Omega\to\infty} P(\Gamma \neq 0) \begin{cases} > 0 & \textit{metal} \\ = 0 & \textit{insulator} \end{cases}$$

How to calculate?

non-equilibrium (arbitrary occupations) → Keldysh



Parameters: $I, M^{-1}, \lambda = 1$ *allow to select the most relevant series*



Self Consistent Born Approximation

Nonlinear integral equation with random coefficients

after standard simple tricks:

Decay due to tunneling

$$\Gamma_l(\epsilon) = \Gamma_l^{(el)}(\epsilon) + \Gamma_l^{(in)}(\epsilon) + \eta;$$

$$\Gamma_l^{(el)}(\epsilon, \boldsymbol{\rho}) = \pi I^2 \delta_\zeta^2 \sum_{l_1, \boldsymbol{a}} A_{l_1}(\epsilon, \boldsymbol{\rho} + \boldsymbol{a})$$

Decay due to e-h pair creation

$$\Gamma_l^{(in)}(\epsilon) = \pi \lambda^2 \delta_\zeta^2 \sum_{l_1, l_2, l_3} Y_{l_1, l_2}^{l_3, l} \int d\epsilon_1 d\epsilon_2 d\epsilon_3 A_{l_1}(\epsilon_1) A_{l_2}(\epsilon_2) A_{l_3}(\epsilon_3) \delta(\epsilon - \epsilon_1 - \epsilon_2 + \epsilon_3) F^{\Rightarrow}_{l_1, l_2; l_3}(\epsilon_1, \epsilon_2; \epsilon_3);$$

$$A_l(\epsilon) = \frac{\pi^{-1} \Gamma_l(\epsilon)}{[\epsilon - \xi_l]^2 + [\Gamma_l(\epsilon)]^2}$$

$$Y_{l_1, l_2}^{l_3, l} \equiv \frac{1}{2} \left[\Upsilon\left(\frac{\xi_{l_2} - \xi_l}{\delta_\zeta}\right) \Upsilon\left(\frac{\xi_{l_1} - \xi_{l_3}}{\delta_\zeta}\right) - \Upsilon\left(\frac{\xi_{l_1} - \xi_l}{\delta_\zeta}\right) \Upsilon\left(\frac{\xi_{l_2} - \xi_{l_3}}{\delta_\zeta}\right) \right]^2$$

$$F^{\Rightarrow}_{l_1, l_2; l_3}(\epsilon_1, \epsilon_2; \epsilon_3) = \frac{1}{4} \left\{ 1 + n_{l_1}(\epsilon_1) n_{l_2}(\epsilon_2) - n_{l_3}(\epsilon_3) \left[n_{l_1}(\epsilon_1) + n_{l_2}(\epsilon_2) \right] \right\};$$

+ kinetic equation for occupation function $n_l(\epsilon)$

Stability of the insulating phase: NO spontaneous generation of broadening

- $\Gamma_\alpha(\epsilon) \equiv 0$ **is always a solution**
- $\epsilon \longrightarrow \epsilon + i\eta$ - ***linear*** **stability analysis**:

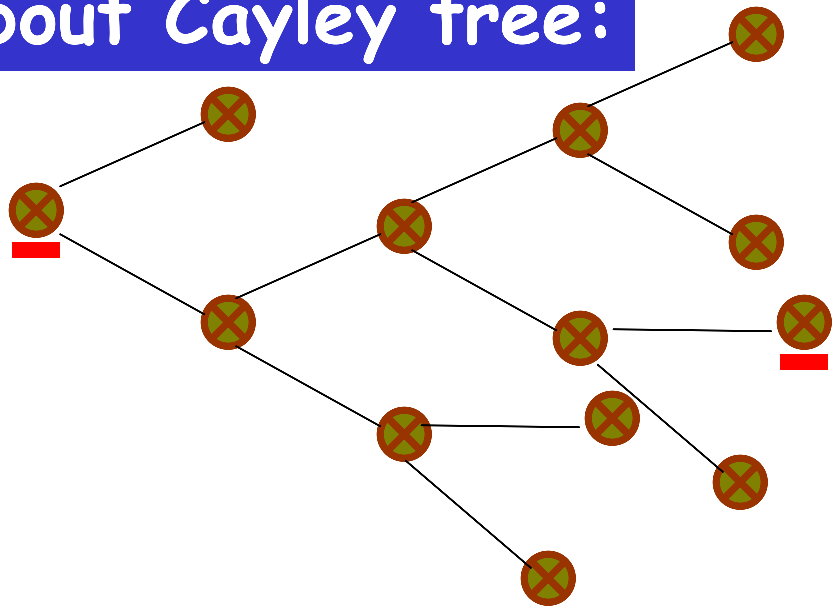
$$\frac{\Gamma}{(\epsilon-\xi_\alpha)^2+\Gamma^2} \to \pi\delta(\epsilon-\xi_\alpha) + \frac{\Gamma}{(\epsilon-\xi_\alpha)^2}$$

- **after n iterations of SCBA equations:**

$$\boxed{P_n(\Gamma) \propto \frac{\eta}{\Gamma^{3/2}}\left(\mathrm{const}\cdot\frac{\lambda T}{\delta_\zeta}\ln\frac{1}{\lambda}\right)^n}$$

first $n \to \infty$
then $\eta \to 0$

About Cayley tree:



Not more than one path between any two sites!

$$n(\rho, j) = \pm 1$$ occupation number

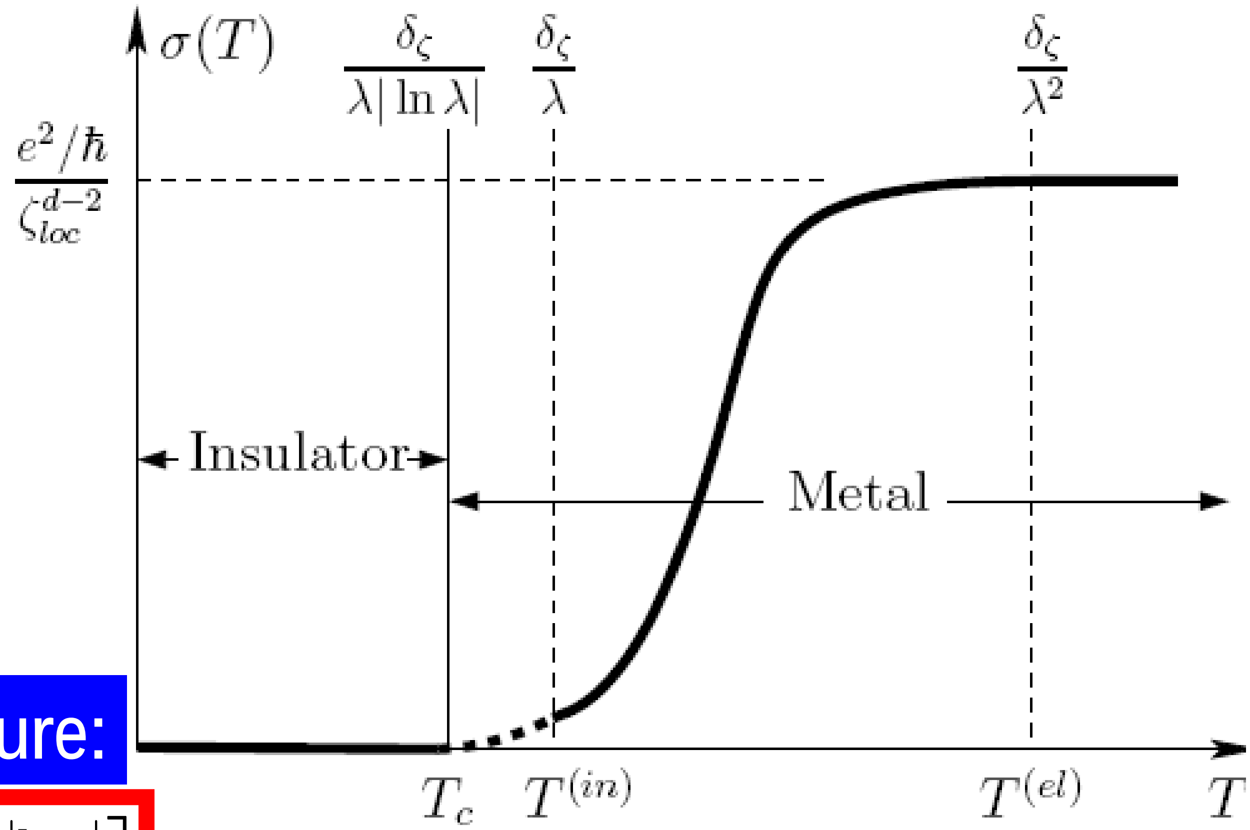
$$\{n(\rho, j)\} \rightarrow \{\tilde{n}(\rho, j)\}$$

Large factorial # of pathes

But # of states gets factorially reduced !

Cancellation?

Yes, but not at the transition point, where there is only one path



Transition temperature:

$$\frac{12\lambda M T_c}{\delta_\zeta}\ln\left(\frac{1}{\lambda}\right) = \exp\left[c\,\frac{\delta_\zeta\,|\ln I|}{T_c}\right]$$

Self-Consistent Born Approximation

Boltzmann Equation

Stability of the metallic phase: Finite broadening is self-consistent

- $$P(\Gamma) = \frac{1}{\sqrt{2\pi\langle\delta\Gamma^2\rangle}} \exp\left[-\frac{(\Gamma - \langle\Gamma\rangle)^2}{2\langle\delta\Gamma^2\rangle}\right]$$

  $\sqrt{\langle\delta\Gamma^2\rangle} \ll \langle\Gamma\rangle$ as long as $\boxed{T \gg \frac{\delta_\zeta}{\lambda}}$

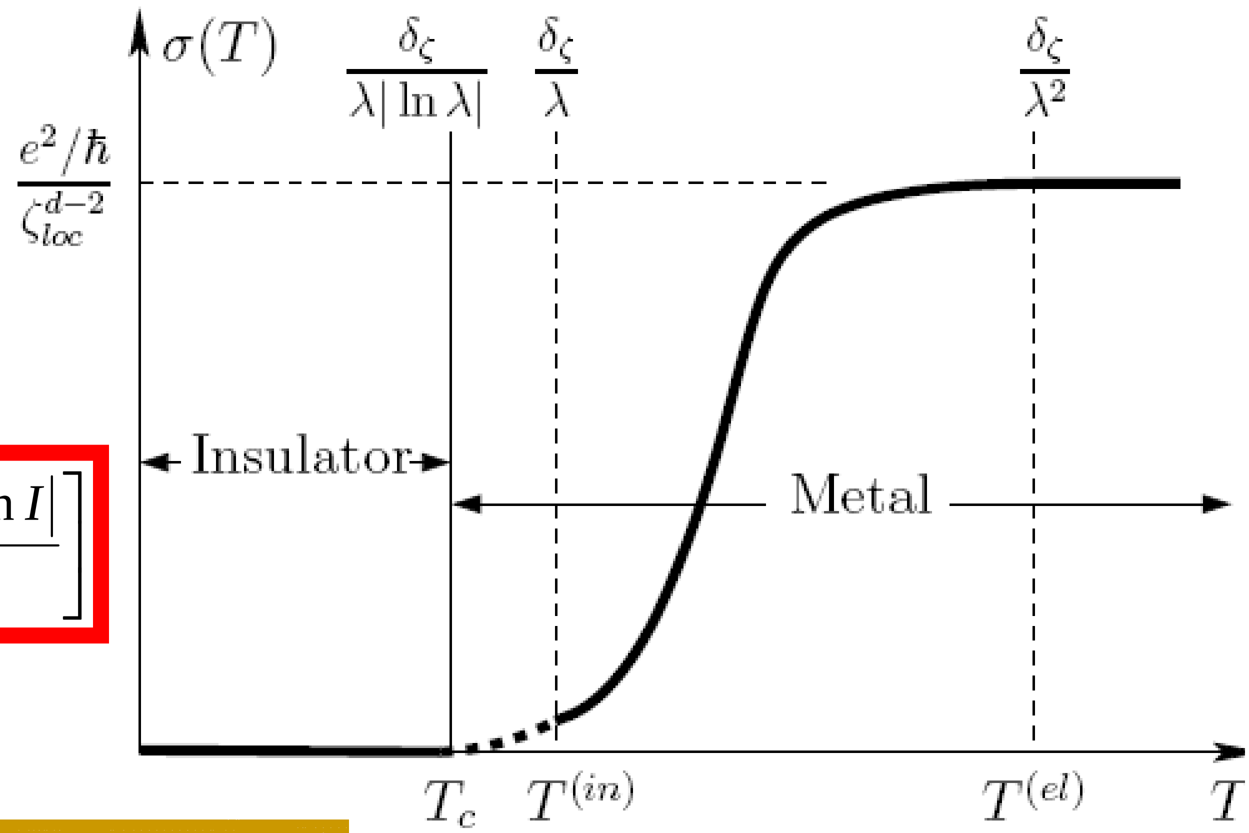
- $\langle\Gamma\rangle \ll \delta_\zeta$ (levels well resolved)

- quantum kinetic equation for transitions between localized states

  $\boxed{\sigma(T) \propto \lambda^2 T^\alpha}$ (model-dependent)

T_c :

$$\frac{12\lambda M T_c}{\delta_\zeta}\ln\left(\frac{1}{\lambda}\right)=\exp\left[\frac{\delta_\zeta\left|\ln I\right|}{T_c}\right]$$



$$T^{(in)}=\frac{\delta_\zeta}{6\pi M\lambda}$$

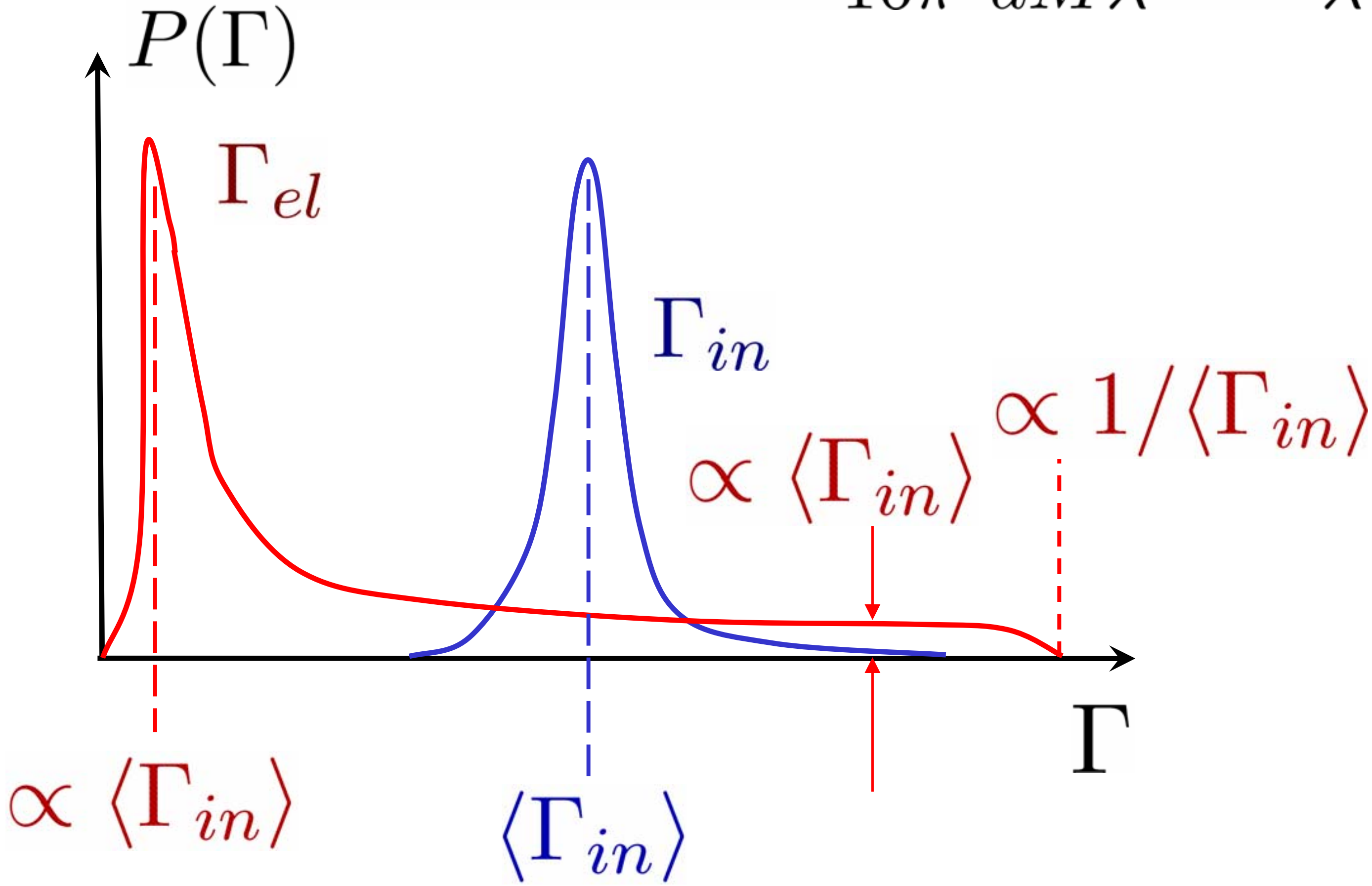
$$T^{(el)}=\frac{\delta_\zeta}{16\pi^2 M d\lambda^2}$$

$$\left\langle\left(\delta\Gamma^{(in)}\right)^2\right\rangle=\left\langle\Gamma^{(in)}\right\rangle^2$$

$$\left\langle\left(\delta\Gamma^{(el)}\right)^2\right\rangle=\left\langle\Gamma^{(el)}\right\rangle^2$$

“Non-ergodic” metal [discussed first in AGKL,97]

$$T_{in} \lesssim T \lesssim T_{el} = \frac{\delta_\zeta}{16\pi^2 dM\lambda^2} \simeq \frac{T_{in}}{\lambda}$$



$$T \ ? \ \ T^{(el)} = \frac{\delta_\zeta}{16\pi^2 M d \lambda^2}$$

$$\sigma(T \gg \sqrt{\delta_\zeta T_{el}}) \approx \sigma_\infty \left(1 - \frac{2}{3}\frac{\delta_\zeta T_{el}}{T^2}\right);$$

$$\kappa(T \gg \sqrt{\delta_\zeta T_{el}}) \approx \kappa_\infty(T)\left[1 - \left(\frac{14}{5} - \frac{24}{\pi^2}\right)\frac{\delta_\zeta T_{el}}{T^2}\right]$$

$$\sigma_\infty \equiv \frac{2\pi e^2 I^2 \zeta_{loc}^{2-d}}{\hbar}, \qquad \kappa_\infty(T) \equiv \frac{2\pi^3 e^2 T I^2 \zeta_{loc}^{2-d}}{3\hbar}.$$

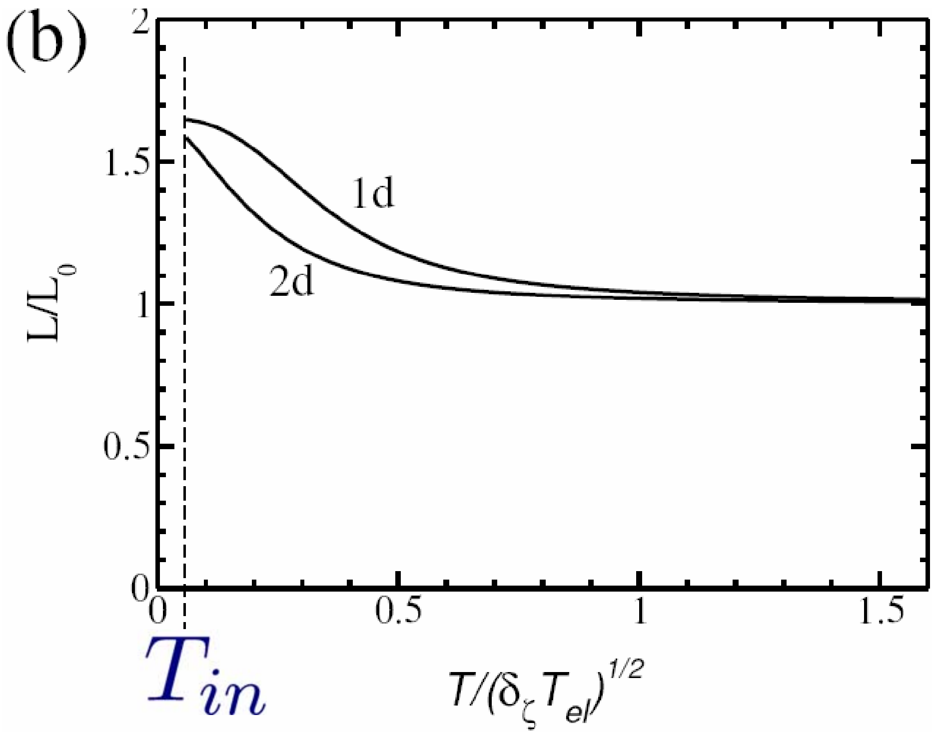
$$T^{el} \ ? \ \ T \ ? \ \ T^{(in)} = \frac{\delta_\zeta}{6\pi M \lambda}$$

$$\sigma(T \ll \sqrt{\delta_\zeta T_{el}}) = \sigma_\infty \frac{\pi}{4}\left(\frac{T^2}{\delta_\zeta T_{el}}\right),$$

$$\kappa(T \ll \sqrt{\delta_\zeta T_{el}}) = \kappa_\infty(T)\frac{48G^2}{\pi^3}\left(\frac{T^2}{\delta_\zeta T_{el}}\right)$$

Lorentz number

$$\frac{\mathrm{L}(T)}{\mathrm{L}_0} \equiv \frac{3e^2\kappa(T)}{\pi^2\sigma(T)T} = \begin{cases} 1 + 0.3\left(\frac{\delta_\zeta T_{el}}{T^2}\right), & T \gg \sqrt{\delta_\zeta T_{el}}, \\ \frac{192\mathrm{G}^2}{\pi^4} \approx 1.65\ldots, & T \ll \sqrt{\delta_\zeta T_{el}}. \end{cases}$$



Many-body mobility edge

T

Large E_k ⇒ high T: extended states

interaction $\longrightarrow$ dephasing $\longrightarrow$ cutoff of WL
(good metal)

δ_ζ/λ^2

Fermi Golden Rule
hopping (bad metal)

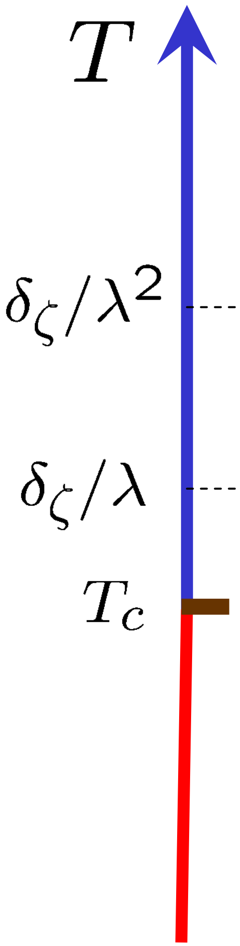
δ_ζ/λ

T_c

transition $\longrightarrow$ mobility edge

Why no activation?

Many-body mobility edge



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δ_ζ/λ^2

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hopping (bad metal)

δ_ζ/λ

T_c

transition $\longrightarrow$ mobility edge

$E, E_c \propto$ *volume*

No activation:

$$E_c \propto \frac{T_c^2}{\delta_\zeta \zeta^d} \quad E \propto \frac{T^2}{\delta_\zeta \zeta^d} \quad \times \textit{volume}$$

$$\exp\left(\frac{E(T)-E_c}{T}\right) \xrightarrow[\textit{volume}\to\infty]{} 0$$

Conclusions & Some speculations

Conductivity exactly vanishes at finite temperature. Finite temperature phase transition without any apparent symmetry change!
Is it an ordinary thermodynamic phase transition or low temperature phase is a glass?

We considered weak interaction.
What about strong electron-electron interactions?
Melting of a pined Wigner crystal?

What if we now turn on phonons?
Cascades.
Is conventional hopping conductivity picture ever correct?